

Madras College Maths Department

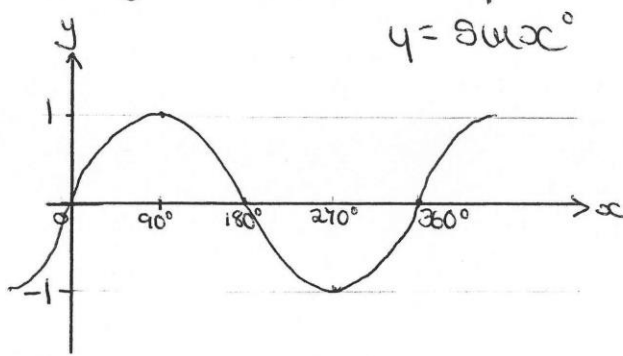
Higher Maths

Solving Trigonometric Equations

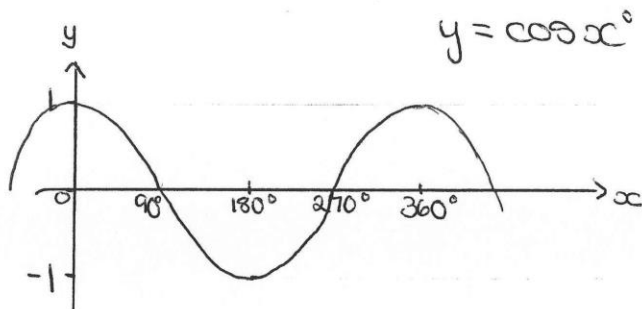
Page	Topic	Textbook
2-3	Solving Trigonometric Equations	8B – 1a, e, 2b, d, f 3, 5. (degree answers) 8D – 1a, c, e, 2a, c, e, g, 3a, c, e, g 4, 6. (radian answers)
4-5	Solving Equations of the Form: $a\cos^2x + b\cos x + c = 0$ and $a\sin^2x + b\sin x + c = 0$	8C – 1a, c, d, 2 a, b 8E – 1a, c 2a, b, c
6	Solving trigonometric equations involving $\sin 2x$ and $\cos 2x$ terms	8F – 2a, c, e, g 3 a, b, c, d 5, 6 8G – 2a, c, 3a
7-9	Solving Equations of the Form: $a\sin x + b\cos x = c$	8H – 1a, b, 4a, 4e
10	Solving Further Trigonometric Equations	8I – 1a, c 2a, b, 3
11	Practice Unit Assessment Questions	
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Graph Recap

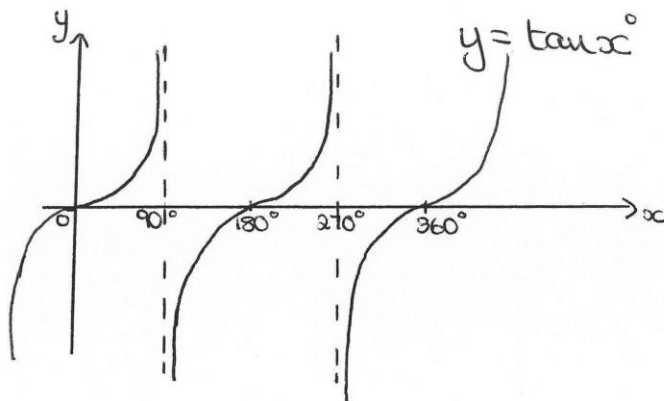
Trigonometry Maths 1 02 + Maths 2 03 (5)

Trig Graphs

Period = 1
Amplitude = 360°



Period = 1
Amplitude = 360°



Period = 180°
Amplitude cannot be measured.

All of the above waves are described as periodic, they consist of a repeated pattern.

Solving Trigonometric Equations of the form: $a \sin (bx - c) + d = e$

Algebraic Solution of Trig Equations

⑦

Examples

Solve for $0 \leq x \leq 360^\circ$

① $\sqrt{2} \sin x^\circ = 0.5$

② $3 \cos 2x^\circ - 1 = -2$

$$\textcircled{3} \quad 4 \tan(x + 45)^\circ = -6$$

$$\textcircled{4} \quad 3 \cos(2x - 120)^\circ = 2$$

$$\textcircled{5} \quad 2 \cos\left(3x - \frac{\pi}{4}\right) = \frac{1}{4} \quad 0 \leq x \leq 2\pi$$

$$6) \quad 3 \tan(2x + 0.2) = 0.5 \quad 0 < x < 2\pi$$

Solving Equations of the Form:

$a\cos^2x + b\cosx + c = 0$ and $a\sin^2x + b\sinx + c = 0$

1) Solve $2\cos^2x - \cosx - 1 = 0$ $0 < x < 2\pi$

2) Solve $3\sin^2x + 8\sinx = 3$ $0 < x < 2\pi$

Trig Equations containing $\sin 2x$ and $\cos 2x$ terms

Examples

$$\textcircled{1} \quad \sin 2x^\circ - \cos x^\circ = 0$$

$$0 \leq x \leq 360^\circ$$

$$\textcircled{2} \quad \cos 2x^\circ + \cos x^\circ = 0$$

$$\textcircled{3} \quad 3\cos 2x^\circ + \sin x^\circ - 2 = 0 \quad 0 < x \leq 360^\circ$$

Solving Equations of the Form: $a\sin x + b\cos x = c$

1) Solve $3\cos x - 2\sin x = 1.3$ $0 < x < 2\pi$

Solving Further Trigonometric Equations

We may be able to use the addition formulae at times to help us solve trigonometric equations.

Solve $3 \sin x \sin 30 - 3 \cos x \cos 30 = 2$ $0 < x < 360$

Practice Unit Assessments

Practice test 1

- 1 Solve $\sqrt{2} \cos 2x^\circ = 1$, for $0^\circ \leq x^\circ \leq 180^\circ$.
- 2 Solve $4 \sin 2t^\circ - \cos t^\circ = 0$, for $0^\circ \leq t^\circ \leq 180^\circ$
- 3 How many solutions does $\cos^2 x = \frac{3}{4}$ have in the interval $\frac{\pi}{4} \leq x \leq 2\pi$

Practice test 2

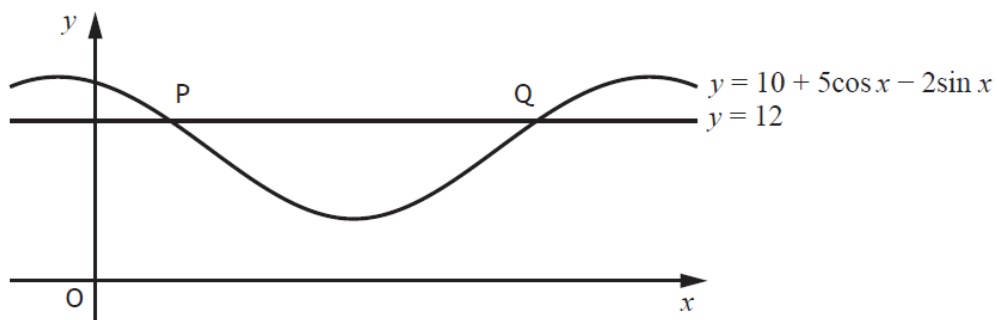
- 1 Solve $2 \cos 2x = \sqrt{3}$, for $0^\circ \leq x \leq 180^\circ$.
- 2 Solve $2 \sin 2w - \cos w = 0$ for $0^\circ \leq t \leq 180^\circ$.
- 3 How many solutions does $\sin^2 x = \frac{1}{2}$ have in the interval $0 \leq x \leq \frac{\pi}{2}$

Homework 2 – Trigonometric Equations

Paper 1 Questions are non-calculator

- 1 (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$,
where $k > 0$ and $0 < a < 2\pi$.
- (b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$
and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



Find the x -coordinates of P and Q.

SQA Higher Maths 2016 Paper 2 Question 8

- 2 Solve the equation

$$\sin x - 2 \cos 2x = 1 \quad \text{for } 0 \leq x < 2\pi. \quad 5$$

SQA Higher Maths 2014 Paper 2 Question 6

- 3 Solve the equation $\sin 2x^\circ = 6\cos x^\circ$ for $0 \leq x \leq 360$. 4

SQA Higher Maths 2007 Paper 1 Question 6

- 4) Solve $2 \cos 2x - 5 \cos x - 4 = 0$ for $0 \leq x < 2\pi$ 5

SQA Higher Maths 2010 Paper 2 Question 4

Solve algebraically the equation

$$\sin 2x = 2 \cos^2 x \quad \text{for } 0 \leq x < 2\pi \quad 6$$

SQA Higher Maths 2013 Paper 2 Question 8

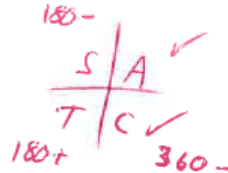
Unit Assessment Practice 1 Solutions

$$\textcircled{1} \quad \sqrt{2} \cos 2x = 1 \quad 0 \leq x \leq 180^\circ$$

$$\cos 2x = \frac{1}{\sqrt{2}}$$

$$\cos^{-1} \frac{1}{\sqrt{2}} = 45^\circ$$

$$2x = 45, 360 - 45, \dots$$



$$2x = 45, 315, 405, 675$$

+360

+360

$$x = \underline{22.5^\circ}, \underline{157.5^\circ}, \cancel{202.5^\circ}, \cancel{337.5^\circ} \quad 0 \leq x \leq 180^\circ$$

$$\textcircled{2} \quad 4 \sin 2t - \cos t = 0 \quad 0 \leq t \leq 180^\circ$$

$$4 \times 2 \sin t \cos t - \cos t = 0$$

$$8 \sin t \cos t - \cos t = 0$$

$$\cos t (8 \sin t - 1) = 0$$

✓

↓

$$\cos t = 0$$

$$8 \sin t - 1 = 0$$

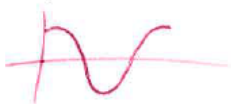
$$8 \sin t = 1$$

$$\sin t = \frac{1}{8}$$



$$t = 7.18, 180 - 7.18$$

$$t = \underline{7.18^\circ}, \underline{172.8^\circ} \rightarrow t = \underline{7.18^\circ}, \underline{90^\circ}, \underline{172.8^\circ}$$



$$t = 90^\circ$$

$$(0 \leq t \leq 180^\circ)$$

$$\textcircled{3} \quad \cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \sqrt{\frac{3}{4}}$$

$$= \pm \frac{\sqrt{3}}{2} \quad (\text{Need to look in all 4 quadrants as both positive \& negative values}).$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \text{ radians}$$

$$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

but $\frac{\pi}{4} \leq x \leq 2\pi$ so only 3 solutions

↓

S	A
T	C

Unit Assessment Practice 2 Solutions

$$\textcircled{1} \quad 2 \cos 2x = \sqrt{3} \quad 0 \leq x \leq 180^\circ$$

$$\cos 2x = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

$$2x = 30, 360 - 30, \dots$$

$$2x = 30, 330, 390, 690$$

$$x = \underline{15^\circ}, \underline{165^\circ}, \underline{195^\circ}, \underline{345^\circ} \quad 0 \leq x \leq 180^\circ$$

$$\textcircled{2} \quad 2 \sin 2\omega - \cos \omega = 0$$

$$2 \times 2 \sin \omega \cos \omega - \cos \omega = 0$$

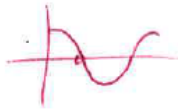
$$\cos \omega (4 \sin \omega - 1) = 0$$

$$\downarrow$$

$$\cos \omega = 0$$

$$\downarrow$$

$$4 \sin \omega - 1 = 0$$

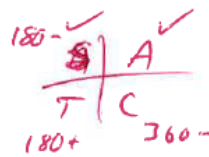


$$\omega = 90^\circ$$

$$4 \sin \omega = 1$$

$$\sin \omega = \frac{1}{4}$$

$$\omega = 14.5^\circ, 165.5^\circ$$



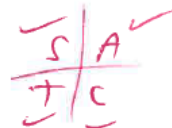
$$\omega = \underline{\underline{14.5^\circ, 90^\circ, 165.5^\circ}}$$

$$\textcircled{3} \quad \sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \sqrt{\frac{1}{2}}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} \quad (\text{Need to look in all 4 quadrants})$$

$$\sin^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$



$$x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$x = \underline{\underline{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}} \quad 0 \leq x \leq \frac{\pi}{2}$$

only one solution in the interval $0 \leq x \leq \frac{\pi}{2}$